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UNSTEADY FREE CONVECTION TRANSIENT FLOW OF AN INCOMPRESSIBLE DISSIPATIVE VISCOUS FLUID

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ABSTRACT

This study deals with the unsteady free convective transient flow of an incompressible dissipative viscous fluid past an infinite vertical plate in presence of viscous dissipative heat, under the influence of a uniform transverse magnetic field. The velocity profiles, temperature profiles, skin friction and rate of heat transfer have been discussed with the help of graphs. The problem is governed by a coupled non-linear system of partial differential equation. The exact solutions are not possible hence explicit finite deference method is employed.

KEYWORDS: *Non-Linear System, Velocity Profiles, Field of Engineering*

INTRODUCTION

The problem of free convection and mass transfer flow of viscous fluid under the influence of magnetic field has attracted the interest of many authors in view of its application to geo-physics, astrophysics and engineering. Boundary layer control in the field of aerodynamics and so on. The researches on flow of viscous fluids are being used in almost all field of Engineering including Bio-medical Science, meteorology, physical chemistry, plasma physics, Monsoon dynamics and allied field. The unsteady conducting free convection flow of dissipative fluids past an infinite plate have received much attention because of non linearity of the governing equations, without taking into account viscous dissipative heat and magnato-hydrodynamic. This problem was discussed by Siegal (1958) by integral method. Some of the literature surveys and review of pertinent work in this field are documented by Goldostein and Eckert (1960), Sparrow and Gregg (1960), Chung and Anderson (1961), Gebhart (1961), Menold and Yang (1962), Schetz and Eichhorn (1962) and Goldestien RJ. and Briggs (1964), In all these problems the effects of viscous dissipative heat and Magnetohydrodynamic was assumed to be neglected but Gebhart (1962) has shown that when the temperature difference is small or in high Prandtl number fluid or when the gravitational field is of high intensity, viscous dissipative heat should be taken into account in steady free convection flow past a semi-infinite vertical plate. The following assumption Soundalgekar, Bhatt and Mohiuddin (1979) discussed the effects of free convection current on the flow past an impulsively started infinite isothermal vertical plate. The free convection and mass transfer effects on the flow past an infinite moving vertical porous plate with constant suction and heat sources when free stream velocity is an oscillatory function of time was presented by Raptis (1982).

The combined buoyancy effect of thermal and mass diffusion of magneto hydro dynamic natural convection flow was presented by Agrawal, Ram and Singh (1980). The free convection heat transfer between two long vertical plates moving in opposite direction was presented by Kajravelu (1978). Recently- Soundalgekar, Lahurikar and Pohanerkar have studied free convection flow of an incompressible viscous dissipative fluid; this problem is governed by coupled non linear system of partial differential equation solved by finite difference technique. Present study is unsteady free convection flow of a dissipative viscous fluid past an infinite vertical plate, on taking into account, the viscous dissipative heat under the influence of uniform transverse magnetic field. This problem is governed now by non-linear coupled differential equations whose exact solution is not possible. So we employ explicit finite difference method for its solution. In present study we consider the unsteady transient free convection flow of an incompressible dissipative viscous fluid past an infinite vertical plate on taking into account viscous dissipative heat under the influence of a uniform transverse magnetic field. The velocity, temperature coefficient of skin friction rate of heat transfer and the effect of magnetic field parameter *M*, the Prandtl number *Pr*, and the Eckert number *E* have been discussed with the help of graph.

LIST OF SYMBOLS USED

x’,y’	→	Cartesian coordinate system	u	→	Velocity of fluid
g	→	Acceleration due to gravity	K	→	Thermal conductivity of fluid
p	→	Density of fluid	v	→	Kinematics Viscosity
β	→	Coefficient of volume expansion	Tω	→	Temperature of plate
T∞	→	Temperature of fluid for away from plate	μ	→	Viscosity of fluid
σ	→	Electrical conductivity of fluid	μe	→	Magnetic permeability
Cp	→	Specific heat at constant pressure	U_0	→	Reference velocity
TR	→	Reference time	L	→	Reference length
θ	→	Dimensional temperature	Pr	→	Prandtl number parameter
E	→	Eckert number parameter	M	→	Magnetic field parameter
q	→	Heat per unit area per unit time	η	→	Dimensionless parameter
H0	→	Magnetic field strength			

FORMULATION OF THE PROBLEM

Consider the unsteady hydrodynamic free convection flow of an incompressible, viscous liquid past an infinite vertical plate. At initially the temperature of the plate and the fluid are assumed to be the same. The time t' > 0, the plate temperature is raised to tω which is then maintained constant under these conditions. The flow-variable are functions of t' and y' alone. The x' -axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to it. A uniform magnetic field of intensity H0 is applied in the y-direction. Therefore the velocity and the magnetic field are given by $\overline{q} = (u, o)$ and $\overline{H} = (0, H_0)$. The fluid being slightly conducting the magnetic Reynolds number is much less than unity and hence the induced magnetic field can be neglected in compression with the applied magnetic field in the absence of any input electric field as Sparrow E.M. and Cess R.D. (1972). Then the governing equation of flow are-

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u}{\partial y'^2} + g\beta(T' - T'_\infty) - \frac{\sigma \mu_e^2 H_0^2}{\rho} u' \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = K \frac{\partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'} \right)^2 \tag{2}$$

boundary condition are -

$$\left.\begin{array}{lllll} t' \le 0, & u' = 0, & T' = T'_\infty & for \quad all & y' \\ t' > 0, & u' = 0, & T' = T'_\omega & at & y' \\ & u' = 0, & T' = T'_\omega & at & y' \to \infty \end{array}\right\} \tag{3}$$

we introduce the following non-dimensional variables -

$$\left.\begin{array}{llll} \Delta T = T' - T'_\omega, & U_0 = (\upsilon g\beta \Delta T)^{113}, & L = \left(\frac{g\beta\Delta T}{v^2} \right)^{\frac{-1}{3}}, & T_R = \left(\frac{g\beta\Delta T}{v} \right)^{\frac{-2}{3}} \\ t = \frac{t'}{T_R}, & y = \frac{y'}{L}, & u = \frac{u'}{U_0}, & \theta = \frac{(T' - T'_\infty)}{(T'_\omega - T'_\infty)}, \\ P_r = \frac{\mu C_p}{K}, & E = \frac{U_0^2}{C_p \Delta T}, & M = \frac{\sigma \mu_e^2 U_0^2 T_R}{\rho} & \end{array}\right\} \tag{4}$$

Using the above variables of equation (4) equations (1) and (2) reduced to-

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \theta - Mu \tag{5}$$

$$P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + \Pr E \left(\frac{\partial u}{\partial y} \right)^2 \tag{6}$$

the boundary condition (3)reduce to –

$$\left.\begin{array}{lllll} t \le 0, & u = 0, & \theta = 0 & for \quad all & y' \\ t > 0, & u = 0, & \theta = 1 & at & y = 0 \\ & u = 0, & \theta \to 0 & at & y \to \infty \end{array}\right\} \tag{7}$$

SOLUTION OF THE PROBLEM

A new dimensionless variable $\eta = \dfrac{y}{1+y}$ (8)

In view of (8) the equation (5) to (7) as

$$\frac{\partial u}{\partial t}=(1-\eta)^4\frac{\partial^2 u}{\partial \eta^2}-2(1-\eta)^3\frac{\partial u}{\partial \eta}+\theta-Mu \tag{9}$$

$$\Pr\frac{\partial \theta}{\partial t}=(1-\eta)^4\frac{\partial^2 \theta}{\partial \eta^2}-2(1-\eta)^3\frac{\partial \theta}{\partial \eta}+(1-\eta)^4\Pr E\left(\frac{\partial u}{\partial \eta}\right)^2 \tag{10}$$

$$\left.\begin{array}{llll} t\le 0, & u=0, & \theta=0 & for\quad all\quad \eta \\ t>0, & u=0, & \theta=1 & at\quad \eta=0 \\ & u=0, & \theta\to 0 & at\quad \eta\to\infty \end{array}\right\} \tag{11}$$

Here equation (9) and (10) are non-linear coupled partial differential equations and are to be solved by using the initial and boundary condition (11). However, exact solutions are not possible for this set of equations and hence we solve these equations by finite-difference method. The equivalent finite difference scheme of equation for (9), (10) and (11) are as follows:

$$\frac{U_{ij+1}-U_{i,j}}{\Delta t}=(1-\eta_{i,j})^4\left\{\frac{U_{i+1j}-2U_{i,j}+U_{i-1,j}}{(\Delta\eta)^2}\right\}-2(1-\eta_{i,j})^3\left\{\frac{U_{i+1,j}-U_{i,j}}{\Delta\eta}\right\}+\theta_{i,j}-MU_{i,j} \tag{12}$$

$$\Pr\frac{\theta_{ij+1}-\theta_{i,j}}{\Delta t}=(1-\eta_{i,j})^4\left\{\frac{\theta_{i+1j}-2\theta_{i,j}+\theta_{i-1,j}}{(\Delta\eta)^2}\right\}-2(1-\eta_{i,j})^3\left\{\frac{\theta_{i+1,j}-\theta_{i,j}}{\Delta\eta}\right\}+(1-\eta_{i,j})^4\Pr E\left\{\frac{U_{i+1,j}-U_{i,j}}{\Delta\eta}\right\}^2 \tag{13}$$

Here index i refers to 17 and j to time. The mesh system is divided by taking Δη = 0.1 from the initial condition in (11) as

$$\left.\begin{array}{llll} u(0,0)=0, & \theta(0,0)=1, & u(i,0)=0, & \theta(i,0)=0 \\ for\quad all & i & except & i=0 \\ u(0,j)=0, & \theta(0,j)=1 & for\quad all & i \\ u(1,j)=0, & \theta(1,j)=0 & for\quad all & i \end{array}\right\} \tag{14}$$

First the velocity at the time step viz. u(i,j+ 1) (i=I,10) is computed from (12) in term of velocity and temperatures at points on the earlier time step. Then θ(i,j+1) is computed from (13). The procedure is repeated until t=l, (i.e. j=800). During computation Δt was chosen as 0.00125. These computations were carried out for Pr(=0.71, 7) and E(=0, 0.1, 0.3, 0.5) to judge the accuracy of the convergence and stability of finite-difference scheme, the same program was run the smaller values of Δt, i.e. Δt = 0.0009, 0.001 and no significant change was observed. Hence we conclude that the finite-difference scheme is stable and convergent.

SKIN FRICTION

Given non-dimensional form as

$$\tau = -\left[\frac{du}{d\eta}\right]_{\eta=0} \quad \text{here} \quad \tau = \frac{\tau'}{\rho U_0^2}$$

Numerical values of r are calculated by applying the Newton's interpolation formula for four points.

HEAT TRANSFER

The non-dimensional form, the rate of heat transfer can be shown to be given by

$$q = -\left[\frac{d\theta}{d\eta}\right]_{\eta=0}$$

We have computed q by following the above procedure. These values of q are listed in table 1.

RESULT AND DISCUSSION

The velocity profit (u) versus (η) for different values of magnetic field parameter M and time t respectively for cases of the Prandtl number Pr (=0.71 and 7.00). Then we see that the velocity profile (u) increases with increase in time t, where as velocity (u) decreases with the increase in M, in all the cases Pr (=0.71 and 7.00). Hence we conclude that velocity (u) decrease with increase in Prandtl number also we observed that velocity (u) increases first near the plate and then trend gets reversed as η increase which is shown in Fig (1) and (2). The Temperature distribution θ is drawn against 17 for different value of Pr and 1. We see that θ decreases with increase in Pr {(3), (4), (5)}. Further, we observe that θ increase in t {(I), (2), (3)}, here also we conclude that θ decrease with increase in 17 which is shown in Fig (3). The skin-friction is shown against t for different value of M in the cases of Pr (=0.71 and 7.00) respectively. Here we observed the T decreases with increase in M or Pr where as it increases with the increase in t which is shown in Fig (4). We observe that q increases with the increases in M where as it decreases with increase in t. Also we observe that u is drawn against η for different value of Eckert number E in this case velocity (u) increases with increase E for all cases of Prandtl number. Further we observe that q increases with the increase in Pr which is shown in table 1.

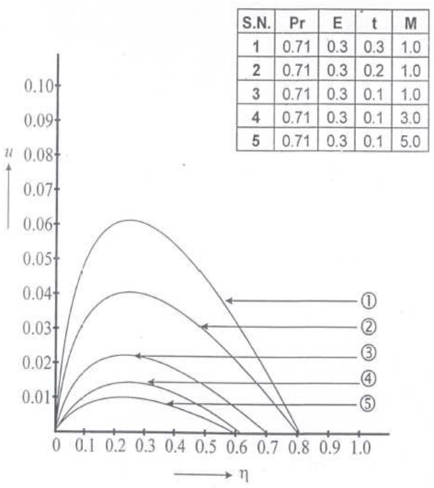

Figure 1: Velocity Profile

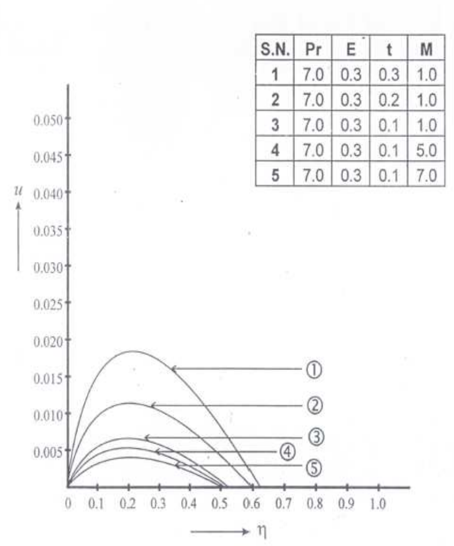

Figure 2: Velocity Profile

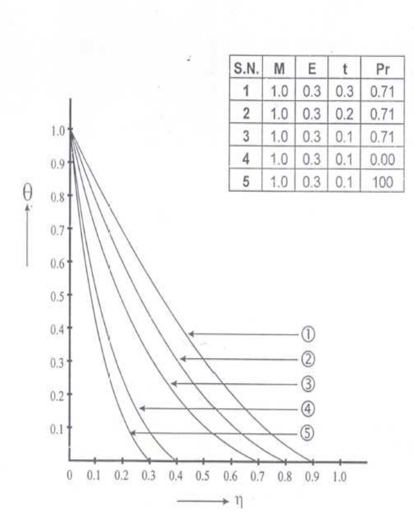

Figure 3: Temperature Profile

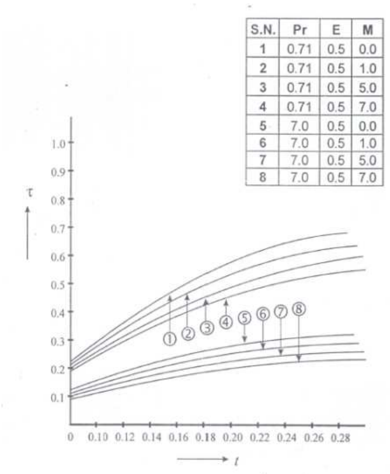

Figure 4: τ Against t for Different M

Table l: Value of q when E=O.3

Pr	t/M	1.0	5.0	7.0
0.71	0.1	1.46691025310025	1.466952321643101	1.46696932150880
	0.2	1.0263125091201	1.02634225069120	1.02640350921620
	0.3	0.60213401961801	0.60314231629002	0.68201509213321
7.0	0.1	4.72351692031120	4.72352931462902	4.72382931624123
	0.2	3.02134296132013	3.02136923497213	3.02213416923812
	0.3	2.01132945612312	2.01142936916654	2.01182345618213
100.0	0.1	15.93215628932126	15.932172931461123	15.93221345216113
	0.2	14.90235436891234	14.902376542131234	14.90247532162214
	0.3	13.30506921354278	13.30508963125981	13.30507921462293

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